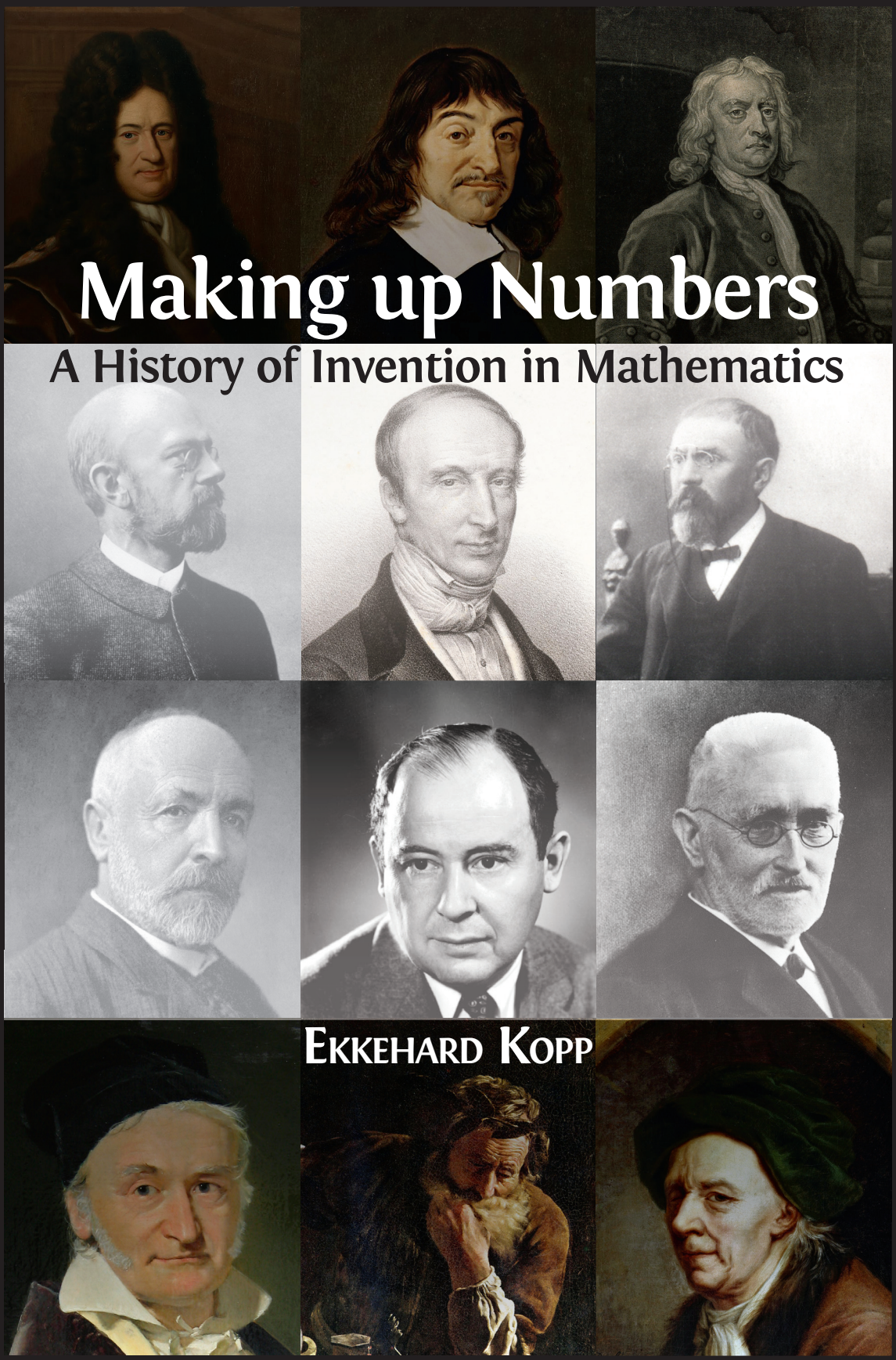

Making up Numbers

A History of Invention in Mathematics

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Epilogue

I will leave matters there. Other extensions of the real number system have been proposed—see [**28**] for a striking example. By starting with counting and linking this to notions of the 'number line', we have seen how mathematicians have extended the number concept progressively over the centuries. Infinitesimals, it seems, are not as easily banished into the outer darkness as Weierstrass and others had supposed—but they had to change their clothes significantly in order to appear more respectable!

That said, what David Hume, who so clearly abhorred 'horn angles' (see the end of **Chapter 5**), might have thought about the infinitely many different orders of infinitesimals now made possible, must remain an open question. And it is easy to imagine how Kronecker (and, no doubt, Brouwer) might have reacted to the hyperreals. We might counter their concerns with Hadamard's confident assertion (quoted in Section 1) that *'essential progress in mathematics'* results from including notions which, for earlier generations, *'were "outside mathematics" because it was impossible to define them'.* And, after all, not even Plato's Olympian edicts stopped Archimedes from employing *neusis* constructions – nor, indeed, infinitesimal slices!

The present mathematical community has largely taken hyperreals in its stride, while seldom showing great interest in the details. One reason is the existence of a meta-theorem that maintains (roughly speaking) that any result which can be proved by nonstandard methods also has a 'standard' proof–which may well be rather longer, however! In this sense, the practitioners of nonstandard analysis appear to be closer to the current 'mainstream' than is the group at the opposite end of the spectrum, the constructivists, who not only reject infinitesimals, but also restrict the real numbers they accept to the numbers (essentially) definable in finitely many words.

Neither of these opposite poles has attracted more than a fairly small minority of practitioners to date. In both cases the 'entry fee' to participation, having to learn radically new techniques and adopt unfamiliar perspectives, may seem quite high to many researchers, trained as they usually are in techniques and subject matter still dominated by the groundwork laid in the late nineteenth century. Whether and how this may change only time will tell.

Let us therefore leave the last word to the venerable Sir Francis Bacon.

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Etiam capillus unus habet umbram suam. (The smallest hair casts a shadow.)

Sir Francis Bacon, *Ornamenta Rationalia, or, Elegant Sentences*, 1625.