Models in Microeconomic Theory

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Models in Microeconomic Theory covers basic models in current microeconomic theory. Part I (Chapters 1–7) presents models of an economic agent, discussing abstract models of preferences, choice, and decision making under uncertainty, before turning to models of the consumer, the producer, and monopoly. Part II (Chapters 8–14) introduces the concept of equilibrium, beginning, unconventionally, with the models of the jungle and an economy with indivisible goods, and continuing with models of an exchange economy, equilibrium with rational expectations, and an economy with asymmetric information. Part III (Chapters 15–16) provides an introduction to game theory, covering strategic and extensive games and the concepts of Nash equilibrium and subgame perfect equilibrium. Part IV (Chapters 17–20) gives a taste of the topics of mechanism design, matching, the axiomatic analysis of economic systems, and social choice.

The book focuses on the concepts of model and equilibrium. It states models and results precisely, and provides proofs for all results. It uses only elementary mathematics (with almost no calculus), although many of the proofs involve sustained logical arguments. It includes about 150 exercises.

With its formal but accessible style, this textbook is designed for undergraduate students of microeconomics at intermediate and advanced levels.

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19 Socialism

Consider a society in which each individual can produce the same consumption good, like food, using a single input, like land. Each individual is characterized by his productivity. The higher an individual's productivity, the more output he produces with any given amount of the input.

An economic system can be thought of as a rule that specifies the output produced by the entire society and the allocation of this output among the individuals as a function of the individuals’ productivities. Should individuals with high productivity get more output than ones with low productivity? Should two individuals with the same productivity receive the same amount of output? Should an increase in an individual's productivity result in his receiving more output? The design of an economic system requires an answer to such questions.

The approach in this chapter (like those in Chapters 3 and 20) is axiomatic. The central result specifies conditions capturing efficiency and fairness that are satisfied only by an economic system that resembles the socialist ideal.

19.1 Model

A society contains \( n \) individuals, denoted \( 1, \ldots, n \). A fixed amount of a single good, referred to as input (like land), is available for division among the individuals. For convenience, we take this amount to be 1. Each individual uses input to produce output, which we refer to as wealth. Each individual \( i \) is characterized by his productivity \( \lambda_i \geq 0 \); using an amount \( \alpha_i \) of the input, he produces the amount \( \alpha_i \lambda_i \) of wealth.

**Definition 19.1: Society and productivity profile**

A society consists of a set of individuals \( N = \{1, \ldots, n\} \) and a total amount of input available, which we assume to be 1.

A productivity profile is a vector \( (\lambda^1, \ldots, \lambda^n) \) of nonnegative numbers. For any nonnegative number \( \alpha^i \), individual \( i \) transforms the amount \( \alpha^i \) of input into the amount \( \alpha^i \lambda^i \) of wealth.

A distribution of the input is a vector \( (\alpha^1, \ldots, \alpha^n) \) of nonnegative numbers with sum 1: \( \sum_{i=1}^{n} \alpha^i = 1 \). If each individual \( i \) is assigned the amount \( \alpha^i \) of the input,
then the total wealth of the society is \( \sum_{i=1}^{n} \alpha^i \lambda^i \), which has to be divided among
the individuals.

**Definition 19.2: Feasible wealth profile**

For any productivity profile \((\lambda^1, \ldots, \lambda^n)\), a profile \((w^1, \ldots, w^n)\) of nonnegative numbers is a feasible wealth profile if for some profile \((\alpha^1, \ldots, \alpha^n)\) of nonnegative numbers with \(\sum_{i=1}^{n} \alpha^i = 1\) we have \(\sum_{i=1}^{n} w^i = \sum_{i=1}^{n} \alpha^i \lambda^i\).

We model an economic system as a rule that specifies for each productivity profile a feasible wealth profile. Note that this notion of an economic system does not specify the amount of wealth each individual produces; the same wealth profile may be achieved by different distributions of the input among the individuals.

**Definition 19.3: Economic system**

An economic system is a function \(F\) that assigns to every productivity profile \((\lambda^1, \ldots, \lambda^n)\) a feasible wealth profile \(F(\lambda^1, \ldots, \lambda^n)\).

If \(F(\lambda^1, \ldots, \lambda^n) = (w^1, \ldots, w^n)\) then we write \(F^i(\lambda^1, \ldots, \lambda^n) = w^i\), the wealth assigned to individual \(i\) by the rule \(F\) given the productivity profile \((\lambda^1, \ldots, \lambda^n)\). Notice that an economic system is a rule that specifies how wealth is distributed for every possible productivity profile, not only for a specific profile.

The distribution of the input determines the total wealth produced. The maximal wealth is obtained by assigning all the input to the individuals with the highest productivity. (If no two individuals have the same productivity, that means that all the input is assigned to the most productive individual.) The equal distribution of the input among all individuals is typically inefficient.

One alternative for distributing the wealth produced by the individuals is to allow each individual to keep the amount he produces. At the other extreme, the total amount of wealth produced is distributed equally among the individuals. If we combine each of these two rules with each of the two rules for assigning input discussed in the previous paragraph, we get the following four examples of economic systems.

**Example 19.1: Equality of input, no redistribution**

The input is divided equally among the individuals and each individual receives the wealth he produces. That is, for any productivity profile \((\lambda^1, \ldots, \lambda^n)\) we have \(F^i(\lambda^1, \ldots, \lambda^n) = \lambda^i / n\) for each individual \(i\).
Example 19.2: Input to most productive, no redistribution

The input is divided equally among the individuals with the highest productivity and each individual receives the wealth he produces. That is, for any productivity profile \((\lambda^1, \ldots, \lambda^n)\) we have

\[
F^i(\lambda^1, \ldots, \lambda^n) = \begin{cases} 
\lambda^i & \text{if } \lambda^i \geq \lambda^k \text{ for all } k \in N \\
0 & \text{otherwise.}
\end{cases}
\]

Example 19.3: Equality of input, equality of wealth

The input is divided equally among the individuals and the total wealth is also divided equally among the individuals. That is, for any productivity profile \((\lambda^1, \ldots, \lambda^n)\) we have \(F^i(\lambda^1, \ldots, \lambda^n) = \sum_{j=1}^n \lambda^j / n^2\) for each individual \(i\).

The last of the four combinations of input and wealth allocation rules assigns the input to the most productive individuals and divides the resulting wealth equally among all individuals. It is one possible formalization of the socialist principle “from each according to his ability, to each according to his needs” (Marx 1971, Section I.3) in the case that every individual has the same needs.

Example 19.4: Input to most productive, equality of wealth (socialism)

The input is divided equally among the individuals with the highest productivity and the wealth is divided equally among all individuals. That is, for any productivity profile \((\lambda^1, \ldots, \lambda^n)\) we have \(F^i(\lambda^1, \ldots, \lambda^n) = \max\{\lambda^1, \ldots, \lambda^n\} / n\) for each individual \(i\).

Example 19.5: One worker, one beneficiary

For some individuals \(j_1\) and \(j_2\), individual \(j_1\) is assigned all the input and individual \(j_2\) gets all the wealth. That is, for any productivity profile \((\lambda^1, \ldots, \lambda^n)\) we have

\[
F^i(\lambda^1, \ldots, \lambda^n) = \begin{cases} 
\lambda^{j_1} & \text{if } i = j_2 \\
0 & \text{otherwise.}
\end{cases}
\]

This economic system can be thought of as a reflection of an extreme power relation in which one individual is a master and the other is a slave.
Chapter 19. Socialism

Example 19.6: Input to most productive, wealth relative to productivity

The input is divided equally among the individuals with the highest productivity and each individual receives an amount of wealth proportional to his productivity. That is, for any productivity profile \((\lambda^1, \ldots, \lambda^n)\) we have

\[ F^i(\lambda^1, \ldots, \lambda^n) = \max\{\lambda^1, \ldots, \lambda^n\} \lambda^i / \sum_{j=1}^n \lambda^j \]

for each individual \(i\).

19.2 Properties of economic systems

Here are some properties that an economic system might satisfy. The first property is similar to Pareto stability.

Efficiency An economic system is efficient if, for every productivity profile, no feasible wealth profile different from the one generated by the system is better for some individuals and not worse for any individual.

Definition 19.4: Efficient economic system

An economic system \(F\) is efficient if for every productivity profile \((\lambda^1, \ldots, \lambda^n)\) there is no feasible wealth profile \((w^1, \ldots, w^n)\) for which \(w^i \geq F^i(\lambda^1, \ldots, \lambda^n)\) for all \(i = 1, \ldots, n\), with at least one strict inequality.

An economic system is efficient if and only if, for every productivity profile, the sum of the wealths the system assigns to the individuals is equal to the maximum total wealth that can be produced. The maximum total wealth is produced only if the input is distributed among the individuals with the highest productivity, so among the examples in the previous section, only the economic systems in Examples 19.2, 19.4, and 19.6 are efficient.

Symmetry An economic system is symmetric if for any productivity profile in which two individuals have the same productivity, they are assigned the same wealth.

Definition 19.5: Symmetric economic system

An economic system \(F\) is symmetric if for any individuals \(i\) and \(j\) and any productivity profile \((\lambda^1, \ldots, \lambda^n)\),

\[ \lambda^i = \lambda^j \Rightarrow F^i(\lambda^1, \ldots, \lambda^n) = F^j(\lambda^1, \ldots, \lambda^n). \]

Note that this property does not constrain the wealths assigned by the economic system to productivity profiles in which all individuals’ productivities differ. The
property is satisfied by all the examples of economic systems in the previous section except Example 19.5 (one worker, one beneficiary).

Relative monotonicity  An economic system is relatively monotonic if whenever individual $i$ has higher productivity than $j$, the amount of wealth assigned to $i$ is at least as high as the amount assigned to $j$.

**Definition 19.6: Relatively monotonic economic system**

An economic system $F$ is relatively monotonic if for every productivity profile $(\lambda^1, \ldots, \lambda^n)$,

$$
\lambda^i \geq \lambda^j \implies F^i(\lambda^1, \ldots, \lambda^n) \geq F^j(\lambda^1, \ldots, \lambda^n).
$$

All the examples in the previous section except Example 19.5 (one worker, one beneficiary) satisfy this property.

The properties we have defined, efficiency, symmetry, and relative monotonicity, require that for each productivity profile, the wealths assigned by the economic system satisfy certain conditions. The properties we now define have a different logical structure: they impose conditions on the relation between the wealth distributions assigned by the economic system to different productivity profiles.

**Anonymity**  An economic system is anonymous if it does not discriminate among individuals on the basis of their names. (It may still discriminate among individuals according to their productivity.) Recall that a permutation of the set of individuals $N$ is a one-to-one function from $N$ to $N$. For example, there are six permutations of $N = \{1, 2, 3\}$; one of them is the function $\sigma$ defined by $\sigma(1) = 3$, $\sigma(2) = 2$, and $\sigma(3) = 1$.

Consider a productivity profile $(\lambda^1, \ldots, \lambda^n)$. Given a permutation $\sigma$ of $N$, we consider the new productivity profile in which each individual $i$ has the productivity of individual $\sigma(i)$ in $(\lambda^1, \ldots, \lambda^n)$. That is, we consider the productivity profile $(\lambda^{\sigma(1)}, \ldots, \lambda^{\sigma(n)})$. The anonymity condition requires that the wealth of $i$ for the new productivity profile is the same as the wealth of $\sigma(i)$ in the original profile: $F^i(\lambda^{\sigma(1)}, \ldots, \lambda^{\sigma(n)}) = F^{\sigma(i)}(\lambda^1, \ldots, \lambda^n)$.

**Definition 19.7: Anonymous economic system**

An economic system $F$ is anonymous if for every productivity profile $(\lambda^1, \ldots, \lambda^n)$ and every permutation $\sigma$ of $N$ we have

$$
F^i(\lambda^{\sigma(1)}, \ldots, \lambda^{\sigma(n)}) = F^{\sigma(i)}(\lambda^1, \ldots, \lambda^n) \text{ for all } i = 1, \ldots, n.
$$
All the examples in the previous section except Example 19.5 (one worker, one beneficiary) satisfy this property.

If an economic system is anonymous then it is symmetric (Problem 1a). But anonymity is stronger than symmetry. Symmetry relates only to the way that the economic system assigns wealth to a productivity profile in which some individuals have the same productivity. Anonymity requires also consistency in the wealths assigned for pairs of productivity profiles that are permutations of each other. Suppose, for example, that $N = \{1,2\}$ and consider the economic system that for any productivity profile $(\lambda^1, \lambda^2)$ with $\lambda^1 = \lambda^2$ assigns each individual $i$ the wealth $\frac{1}{2}\lambda^i$ and for any other productivity profile assigns individual 1 the wealth max{$\lambda^1, \lambda^2$} and individual 2 the wealth 0. This economic system is symmetric but not anonymous.

**Monotonicity in own productivity** An economic system is monotone in own productivity if, when an individual’s productivity increases, his wealth does not decrease. Notice the difference between this property and relative monotonicity, which requires that if one individual’s productivity is at least as high as another’s then his wealth is also at least as high.

**Definition 19.8: Monotonicity in own productivity**

An economic system $F$ is monotone in own productivity if for every productivity profile $(\lambda^1, \ldots, \lambda^n)$, every individual $i$, and every number $\Delta > 0$, we have $F^i(\lambda^1, \ldots, \lambda^i + \Delta, \ldots, \lambda^n) \geq F^i(\lambda^1, \ldots, \lambda^i, \ldots, \lambda^n)$.

All the examples in the previous section satisfy this property. If an economic system is monotone in own productivity and symmetric then it is relatively monotonic (Problem 1b).

**Monotonicity in others’ productivities** An economic system is monotone in others’ productivities if any increase in the productivity of one individual does not hurt another individual.

**Definition 19.9: Monotonicity in others’ productivities**

An economic system $F$ is monotone in others’ productivities if for every productivity profile $(\lambda^1, \ldots, \lambda^n)$, any two individuals $i$ and $j$, and every number $\Delta > 0$, we have $F^j(\lambda^1, \ldots, \lambda^i + \Delta, \ldots, \lambda^n) \geq F^j(\lambda^1, \ldots, \lambda^i, \ldots, \lambda^n)$.

All examples in the previous section except Examples 19.2 (input to most productive, no redistribution) and 19.6 (input to most productive, wealth relative to productivity) satisfy this property.
19.3 Characterization of socialism

We now show that socialism is the only economic system that satisfies four of the properties defined in the previous section.

**Proposition 19.1: Characterization of socialist economic system**

The only economic system that is efficient, symmetric, monotone in own productivity, and monotone in others’ productivities is socialism, according to which all wealth is produced by the individuals with the highest productivity and the total wealth is divided equally among all individuals.

**Proof**

Problem 1c asks you to verify that the socialist economic system satisfies the four properties. We now show that it is the only economic system that does so.

Let $F$ be an economic system satisfying the four properties. We first show that for every productivity profile $(\lambda^1, \ldots, \lambda^n)$ and every individual $j$, the wealth $F^j(\lambda^1, \ldots, \lambda^n)$ assigned by $F$ to $j$ is at most $M/n$, where $M = \max\{\lambda^1, \ldots, \lambda^n\}$. Suppose to the contrary that for some productivity profile $(\lambda^1, \ldots, \lambda^n)$, we have $F^i(\lambda^1, \ldots, \lambda^n) > M/n$ for some individual $i$. Given that $M \geq \lambda^j$ for every individual $j$, the repeated application of the monotonicity of $F$ in others’ productivities yields

$$F^i(M, \ldots, M, \lambda^i, M, \ldots, M) \geq F^i(\lambda^1, \ldots, \lambda^n).$$

Now use the monotonicity of $F$ in own productivity to conclude that

$$F^i(M, \ldots, M) \geq F^i(\lambda^1, \ldots, \lambda^n).$$

Let $F^i(M, \ldots, M) = H$. By symmetry, $F^j(M, \ldots, M) = H$ for every individual $j$. But $F^i(\lambda^1, \ldots, \lambda^n) > M/n$, so $H > M/n$, and hence the wealth distribution $(H, \ldots, H)$ is not feasible. Thus $F^i(\lambda^1, \ldots, \lambda^n) \leq M/n$ for every individual $i$.

By efficiency, the input is distributed among the individuals with the highest productivity, so that for every productivity profile $(\lambda^1, \ldots, \lambda^n)$ we have $\sum_{i=1}^n F^i(\lambda^1, \ldots, \lambda^n) = M$ and hence $F^i(\lambda^1, \ldots, \lambda^n) = M/n$ for every individual $i$.

We close the chapter by showing that the four properties are independent in the sense that none of them is implied by the other three. For each property we find an economic system that does not satisfy that property but satisfies
the other three. Thus all four properties are required to reach the conclusion of Proposition 19.1.

**Proposition 19.2: Independence of properties**

The properties of efficiency, symmetry, monotonicity in own productivity, and monotonicity in others' productivities are independent. That is, for each property there is an economic system that does not satisfy that property but satisfies the other three.

**Proof**

**Efficiency**

The economic system defined by \( F^i(\lambda^1, \ldots, \lambda^n) = \min \{\lambda^1, \ldots, \lambda^n\} / n \) satisfies all axioms but efficiency. (For any productivity profile \((\lambda^1, \ldots, \lambda^n)\) that is not constant, \((w, \ldots, w)\) with \(w = \max \{\lambda^1, \ldots, \lambda^n\} / n\) is a feasible wealth profile, and \(w > \min \{\lambda^1, \ldots, \lambda^n\} / n = F^i(\lambda^1, \ldots, \lambda^n)\).)

**Symmetry**

The economic system defined by \( F^1(\lambda^1, \ldots, \lambda^n) = \max \{\lambda^1, \ldots, \lambda^n\} \) and \( F^i(\lambda^1, \ldots, \lambda^n) = 0 \) for \( i \neq 1 \) for every productivity profile \((\lambda^1, \ldots, \lambda^n)\) (all wealth goes to individual 1) satisfies all the axioms except symmetry.

**Monotonicity in own productivity**

The economic system according to which the individuals with the lowest productivity share \( \max \{\lambda^1, \ldots, \lambda^n\} \) equally and the wealth of every other individual is zero satisfies all the axioms except monotonicity in own productivity. (If the productivity of an individual increases from \( \min \{\lambda^1, \ldots, \lambda^n\} \) to a larger number, the wealth assigned to him decreases to zero.)

**Monotonicity in others’ productivities**

The economic system according to which the individuals with the highest productivity share \( \max \{\lambda^1, \ldots, \lambda^n\} \) equally and the wealth of every other individual is zero satisfies all the axioms except monotonicity in others’ productivities. (If the productivity of an individual \( i \) increases from less than \( \max \{\lambda^1, \ldots, \lambda^n\} \) to more than this number, the wealth of every individual whose productivity was formerly \( \max \{\lambda^1, \ldots, \lambda^n\} \) decreases to zero.)

**Comments**

This chapter is not an argument for or against socialism. Its main aim is to demonstrate that some economists are interested in economic systems with a
centralized component and consider fairness to be a criterion by which a system should be judged. The main result shows that a system we label “socialism” is characterized by four properties. Of course other economic systems are characterized by other sets of properties. Those characterizations can help us normatively evaluate economic systems.

In the model the distribution of output depends only on the profile of productivities. No other factors are taken into account; in particular, information about the individuals’ needs is ignored. The model also does not touch upon the issue of incentives, which is central to most economic models. Each individual produces an output proportional to his productivity even if he does not obtain the output.

Problems

1. \textit{Relations between properties}. Show the following results.
   
   \begin{itemize}
   \item \textbf{a. Anonymity implies symmetry.}
   \item \textbf{b.} For $n = 2$, find an economic system that satisfies \textit{monotonicity in own productivity} and \textit{symmetry} but not \textit{relative monotonicity}.
   \item \textbf{c.} The \textbf{socialist economic system} satisfies the properties of \textit{efficiency}, \textit{symmetry}, \textit{monotonicity in own productivity}, and \textit{monotonicity in others’ productivities}.
   \end{itemize}

2. \textit{Input and wealth to most productive}. Consider the economic system $F$ in which the input is divided equally among the individuals with the highest productivity and the wealth is divided equally among these individuals.
   
   \begin{itemize}
   \item \textbf{a.} Show that $F$ satisfies the following \textit{strong monotonicity in own productivity} property. For any productivity profile $(\lambda^1, \ldots, \lambda^n)$ and any number $\Delta > 0$,
     \[
     F^i(\lambda^1, \ldots, \lambda^n) > 0 \quad \Rightarrow \quad F^i(\lambda^1, \ldots, \lambda^i + \Delta, \ldots, \lambda^n) > F^i(\lambda^1, \ldots, \lambda^n).
     \]
   \item \textbf{b.} Show that $F$ is not the only economic system that satisfies strong monotonicity in own productivity, \textit{symmetry}, and \textit{efficiency}.
   \end{itemize}

3. \textit{Shapley value}. Consider economic systems that are \textit{efficient} and \textit{symmetric} and satisfy the following two conditions.
   
   \begin{itemize}
   \item \textbf{Zero contribution}:
     For every \textbf{productivity profile} $(\lambda^1, \ldots, \lambda^n)$ with $\lambda^i = 0$, we have $F^i(\lambda^1, \ldots, \lambda^n) = 0$.
   \end{itemize}
Marginal contribution

Let \((\lambda^1, \ldots, \lambda^n)\) be a profile and \(S\) be a subset of the most productive individuals (those with productivity \(\max\{\lambda^1, \ldots, \lambda^n\}\)). Suppose that the productivity profile \(\mu = (\mu^1, \ldots, \mu^n)\) is obtained from the profile \((\lambda^1, \ldots, \lambda^n)\) by adding \(\delta > 0\) to the productivity of each member of \(S\). Then the wealth of each individual \(j \in S\) is \(F^i(\lambda^1, \ldots, \lambda^n)\) plus the wealth he would receive if the productivity of every individual in \(S\) were \(\delta\) and the productivity of every other individual were 0. Formally, if

\[
(\mu^1, \ldots, \mu^n) = (\lambda^1, \ldots, \lambda^n) + (\Delta^1, \ldots, \Delta^n)
\]

where (i) \(\Delta^i \in \{0, \delta\}\) and (ii) if \(\Delta^i > 0\) then \(\lambda^i = \max\{\lambda^1, \ldots, \lambda^n\}\), then

\[
F^i(\mu^1, \ldots, \mu^n) = F^i(\lambda^1, \ldots, \lambda^n) + F^i(\Delta^1, \ldots, \Delta^n).
\]

(This, for example, \(F^1(8, 8, 7, 6, 4) = F^1(7, 7, 7, 6, 4) + F^1(1, 1, 0, 0, 0)\).)

\(a\). For such an economic system \(F\) find the wealths assigned when \(n = 2\) and the productivity profile is \(1, 3\).

\(b\). For such an economic system \(F\), find the wealths assigned when \(n = 4\) and the productivity profile is \(1, 3, 6, 10\).

\(c\). Imagine that the four individuals with productivity profile \((1, 3, 6, 10)\) arrive on the scene one after the other in one of the 24 possible orders. Consider the marginal contribution of each individual: the increase in the wealth that can be produced due to his arrival. For example, if the order in which the individuals arrive is \((1, 3, 2, 4)\) then the marginal contribution of individual 1 is 1, that of individual 3 is \(6 - 1 = 5\), that of individual 2 is zero (because individual 3, with productivity 6, has already arrived), and that of agent 4 is \(10 - 6 = 4\).

Show that the averages of the marginal contributions are exactly the wealths you found in the previous part. For example, individual 1 makes a positive contribution, and that contribution is 1, only if he is the first in the list. He is first in a quarter of the orders, so the average of his marginal contributions is \(\frac{1}{4}\).

\(d\). For each of the four properties (efficiency, symmetry, zero contribution, and marginal contribution) find an economic system that does not satisfy the property but satisfies the other three.

Notes

This chapter is inspired by the work of John Roemer (for example Roemer 1986).