Models in Microeconomic Theory

Part I (Chapters 1–7) presents models of an economic agent, discussing abstract models of preferences, choice, and decision making under uncertainty, before turning to models of the consumer, the producer, and monopoly. Part II (Chapters 8–14) introduces the concept of equilibrium, beginning, unconventionally, with the models of the jungle and an economy with indivisible goods, and continuing with models of an exchange economy, equilibrium with rational expectations, and an economy with asymmetric information. Part III (Chapters 15–16) provides an introduction to game theory, covering strategic and extensive games and the concepts of Nash equilibrium and subgame perfect equilibrium. Part IV (Chapters 17–20) gives a taste of the topics of mechanism design, matching, the axiomatic analysis of economic systems, and social choice.

The book focuses on the concepts of model and equilibrium. It states models and results precisely, and provides proofs for all results. It uses only elementary mathematics (with almost no calculus), although many of the proofs involve sustained logical arguments. It includes about 150 exercises.

With its formal but accessible style, this textbook is designed for undergraduate students of microeconomics at intermediate and advanced levels.

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2 Choice

2.1 Choice and rational choice

In the previous chapter we discuss an individual's preference relation, a formal concept that describes his mental attitude to all relevant alternatives. We now develop a formal tool to describe an individual's behavior. The two concepts, preferences and choice, are building blocks of the economic models we develop later.

Recall that the notion of a preference relation refers only to the individual's mental attitude, not to the choices he may make. In this chapter, we describe a concept of choice, independently of preferences. This description specifies his decision in any possible choice problem he may confront within the context we are modeling. Suppose, for example, that we want to model a worker who is applying for a job. Then a complete description of his behavior specifies not only which job he chooses if all jobs in the world are open to him, but also his choice from any subset of jobs that he might be offered.

Formally, let $X$ be the set of all the alternatives an individual might face. A choice problem is a nonempty subset $A$ of $X$, from which the individual chooses an alternative. A choice function describes the individual's choice for every possible choice problem.

**Definition 2.1: Choice problem and choice function**

Given a set $X$, a choice problem for $X$ is a nonempty subset of $X$ and a choice function for $X$ associates with every choice problem $A \subseteq X$ a single member of $A$ (the member chosen).

Usually in economics we connect the individual's behavior and his mental attitude by assuming that the individual is rational in the sense that

- he has a preference relation over $X$
- whenever he has to make a choice, he is aware of the set of possible alternatives
- he chooses an alternative that is best according to his preference relation over the set of possible alternatives.
Note that this model of rationality does not make any assumptions about the content of the individual’s preferences. His preferences might be “irrational” in the everyday sense of the word and be inconsistent with what he, or we, would consider to be his well-being. For example, an individual who chooses an alternative that causes him the greatest pain (measured in some way) is rational in the sense we have defined.

If the preference relation of an individual is represented by the utility function $u$, then the individual acts as if he maximizes the function $u$ under the constraint that $x \in A$. Formally we write his problem as

$$\max \{ u(x) : x \in A \}.$$ 

Note that if two individuals have two different strict preference relations and, given any set $A$ choose alternatives in $A$ that are best according to these preference relations, then their corresponding choice functions differ. That is, if for two alternatives $x$ and $y$ one individual prefers $x$ to $y$ and the other prefers $y$ to $x$, then the choice function of the first individual assigns $x$ to the problem $\{x, y\}$ and the choice function of the second individual assigns $y$ to this set.

### 2.2 Rationalizing choice

Human beings usually do not consciously maximize a preference relation when they make decisions. The standard justification for modeling individuals as rational is that although individuals rarely explicitly choose the best alternatives according to their preference relations, their behavior can often be described as if they make choices in this way. Individuals do not have to be aware of their preference relations. The assumption that they maximize some preference relation is appropriate as long as we can describe them as if they behave in this way. Accordingly, we make the following definition.

**Definition 2.2: Rationalizable choice function**

A choice function is rationalizable if there is a preference relation such that for every choice problem the alternative specified by the choice function is the best alternative according to the preference relation.

Notice that this definition requires that the alternative chosen from any set is the unique best alternative. If we were to require only that it is a best alternative, then every choice function would be rationalizable by the preference relation in which all alternatives are indifferent. We return to the issue in Section 5.5.
Let $X = \{a, b, c\}$. The choice function that assigns $a$ to $\{a, b, c\}$, $a$ to $\{a, b\}$, $a$ to $\{a, c\}$, and $b$ to $\{b, c\}$ is rationalized by the preference relation $\succeq$ for which $a \succ b \succ c$. That is, we can describe the behavior of an individual with this choice function as if he always chooses the best available alternative according to $\succeq$.

On the other hand, any choice function that assigns $a$ to $\{a, b\}$, $c$ to $\{a, c\}$, and $b$ to $\{b, c\}$ is not rationalizable. If this choice function could be rationalized by a preference relation $\succeq$, then $a \succ b \succ c$, and $c \succ a$, which contradicts transitivity.

Of the 24 possible choice functions for the case in which $X$ contains three alternatives, only six are rationalizable.

We now give some examples of choice procedures and examine whether the resulting choice functions are rationalizable.

**Example 2.2: The median**

An individual has in mind an ordering of the alternatives in the set $X$ from left to right. For example, $X$ could be a set of political candidates and the ordering might reflect their position from left to right. From any set $A$ of available alternatives, the individual chooses a median alternative. Precisely, if the number of available alternatives is odd, with $a_1 < a_2 < \cdots < a_{2k+1}$ for some integer $k$, the individual chooses the single median $a_{k+1}$, and if the number of alternatives is even, with $a_1 < a_2 < \cdots < a_{2k}$, then the individual chooses $a_k$, the leftmost of the two medians.

No preference relation rationalizes this choice function. Assume that $A$ contains five alternatives, $a_1 < a_2 < a_3 < a_4 < a_5$. From this set, he chooses $a_3$. If he has to choose from $\{a_3, a_4, a_5\}$, he chooses $a_4$. If a preference relation $\succeq$ rationalizes this choice function then $a_3 \succ a_4$ from his first choice and $a_4 \succ a_3$ from his second choice, a contradiction.

Note that the individual’s behavior has a rationale of a different type: he always prefers the central option. But this rationale cannot be described in terms of choosing the best alternative according to a preference relation over the set of available alternatives. The behavior can be rationalized if we view the set of alternatives to be the positions $Y = \{\text{median, one left of median, one right of median, two left of median, two right of median}\}$. Then the first choice problem is $Y$ and the second choice problem is $\{\text{one left of median, median, one right of median}\}$. The
preference relation ≽ given by

median ≻ one left of median ≻ one right of median ≻ …

rationalizes the choice function.

Example 2.3: Steak and salmon

Luce and Raiffa (1957, 288) give an example of a person entering a restaurant in a strange city.

The waiter informs him that there is no menu, but that this evening he may have either broiled salmon at $2.50 or steak at $4.00. In a first-rate restaurant his choice would have been steak, but considering his unknown surroundings and the different prices he elects the salmon. Soon after the waiter returns from the kitchen, apologizes profusely, blaming the uncommunicative chef for omitting to tell him that fried snails and frog’s legs are also on the bill of fare at $4.50 each. It so happens that our hero detests them both and would always select salmon in preference to either, yet his response is “Splendid, I’ll change my order to steak”.

Consider a set $X$ that consists of the four main courses, salmon, steak, snails, and frog’s legs. No preference relation over $X$ rationalizes the person’s behavior, because such a preference relation would have to rank salmon above steak by his choice from \{salmon, steak\} and steak above salmon by his choice from $X$.

A reasonable explanation for the person’s behavior is that although steak appears in both choice problems, he does not regard it to be the same dish. The availability of snails and frog’s legs tells him that the steak is likely to be of high quality. Without this information, he views steak as low quality and chooses salmon.

No preference relation on $X$ rationalizes the person’s behavior, but a preference relation on \{salmon, low quality steak, high quality steak, snails, frog’s legs\} does so:

high quality steak ≻ salmon ≻ low quality steak ≻ snails ≻ frog’s legs.

An underlying assumption behind the concept of a choice function is that an alternative is the same in every choice set in which it appears. The choice function in the example cannot be rationalized because the example identifies two different options as the same alternative.
2.3 Property $\alpha$

We say that a choice function satisfies property $\alpha$ if whenever the choice from $A$ is in a subset $B$ then the alternative chosen from $A$ is chosen also from $B$. We show that (i) any choice function that selects the best alternative according to a preference relation satisfies this property and (ii) any choice function that satisfies the property is rationalizable.

**Definition 2.3: Property $\alpha$**

Given a set $X$, a choice function $c$ for $X$ satisfies property $\alpha$ if for any sets $A$ and $B$ with $B \subset A \subseteq X$ and $c(A) \in B$ we have $c(B) = c(A)$.

Notice that property $\alpha$ is not satisfied by the choice functions in Examples 2.2, 2.3, and 2.4.

**Proposition 2.1: Rationalizable choice function satisfies property $\alpha$**

Every rationalizable choice function satisfies property $\alpha$.

**Proof**

Let $c$ be a rationalizable choice function for $X$ and let $\succeq$ be a preference relation such that for every set $A \subseteq X$, $c(A)$ is the best alternative according to $\succeq$ in $A$. Assume that $B \subset A$ and $c(A) \in B$. Since $c(A) \succeq y$ for all $y \in A$ we have $c(A) \succeq y$ for all $y \in B$ and thus $c(B) = c(A)$. 

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**Example 2.4: Partygoer**

Each of the people in the set $X = \{A, B_1, B_2\}$ organizes a party. A person might be invited to a subset of those parties and can attend only one party. Individuals $B_1$ and $B_2$ are both good friends of the partygoer but the relations between $B_1$ and $B_2$ are tense. The person’s behavior is as follows. If he is invited by $A$ and $B_1$, he accepts $B_1$’s invitation. If he is invited by all three individuals, he accepts $A$’s invitation. He does so because he is worried that accepting the invitation of $B_1$ or $B_2$ will be interpreted negatively by the other individual. Obviously such behavior is not rationalizable by a preference relation over $X$. As in the previous example, the meaning of choosing one alternative ($B_1$) is affected by the presence or absence of another alternative ($B_2$).
Proposition 2.2: Choice function satisfying property α is rationalizable

If X is a finite set then any choice function for X satisfying property α is rationalizable.

Proof

Let c be a choice function for X satisfying property α. Denote by n the number of elements in X. We construct a preference relation that rationalizes c as follows. Denote \( c(X) = a_1 \), \( c(X\{a_1\}) = a_2 \), \( c(X\{a_1,a_2\}) = a_3 \), and so on. That is, \( a_k \) is the choice from the set \( X \) after removing the elements \( a_1, \ldots, a_{k-1} \).

Consider the preference relation \( \succeq \) defined by \( a_1 \succ a_2 \succ \cdots \succ a_n \). Let \( A \) be a choice problem. The best alternative in \( A \) according to \( \succeq \) is the first member of \( A \) in the sequence \( a_1, a_2, \ldots, a_n \), say \( a_m \). By construction, \( c(\{a_m,a_{m+1},\ldots,a_n\}) = a_m \) and since \( A \subseteq \{a_m,a_{m+1},\ldots,a_n\} \) and \( a_m \in A \), from property α we have \( c(A) = a_m \).

2.4 Satisficing

Imagine an employer who must hire a worker. He interviews the candidates in alphabetical order until he reaches a candidate whom he considers to be good enough, and then stops. If no candidate is good enough, he chooses the last candidate to be interviewed.

Formally, denote the set of candidates by \( X \). The employer has in mind a function \( v : X \rightarrow \mathbb{R} \) that measures the candidates' qualities. He has in mind also a number \( v^* \), an aspiration level. Let \( O \) be an ordering of the set \( X \) (for example, alphabetical order), which describes the sequence in which the employer interviews candidates. Given a set \( A \) of alternatives, the employer chooses the first alternative \( a \in A \) in the ordering \( O \) for which \( v(a) \geq v^* \) if such an alternative exists, and otherwise chooses the last element in \( A \) according to \( O \).

Definition 2.4: Satisficing choice function

Let \( X \) be a finite set. Given a function \( v : X \rightarrow \mathbb{R} \) (the valuation function), a number \( v^* \) (the aspiration level), and an ordering \( O \) of \( X \), the satisficing choice function \( c \) is defined as follows. Let \( A = \{a_1, \ldots, a_K\} \) where \( a_1 O a_2 O \cdots O a_K \). Then

\[
c(A) = \begin{cases} 
  a_k & \text{if } v(a_k) \geq v^* \text{ and } v(a_l) < v^* \text{ for } l = 1, \ldots, k-1 \\
  a_K & \text{if } v(a_l) < v^* \text{ for } l = 1, \ldots, K.
\end{cases}
\]
Every alternative \( x \) for which \( v(x) \geq v^* \) is satisfactory and every other alternative is unsatisfactory.

**Proposition 2.3: Satisficing choice function is rationalizable**

A satisficing choice function is rationalizable.

This result can be proved by showing that any satisficing choice function satisfies property \( a \) (see Problem 3). Here we provide a direct proof.

**Proof**

Let \( c \) be the satisficing choice function for valuation function \( v \), aspiration level \( v^* \), and ordering \( O \). We construct a preference relation \( \succeq \) that rationalizes \( c \). At the top of the preference relation we put the satisfactory alternatives, \( X^+ = \{ x \in X : v(x) \geq v^* \} \), in the order given by \( O \). Then we put all the unsatisfactory alternatives, \( X^- = \{ x \in X : v(x) < v^* \} \), in the order given by the reverse of \( O \). (If, for example, \( X = \{ a, b, c, d \} \), \( O \) is alphabetical order, \( v^* = 0 \), and the valuation function is defined by \( v(a) = -1 \), \( v(b) = -2 \), \( v(c) = 1 \), and \( v(d) = 3 \), then the preference relation we construct is \( c \succ d \succ b \succ a \).)

We now show that this preference relation rationalizes \( c \). Let \( A \subseteq X \). If \( A \) contains a member of \( X^+ \) then the best alternative in \( A \) according to \( \succeq \) is the first alternative, according to \( O \), in \( A \cap X^+ \), which is \( c(A) \). If \( A \) does not contain a member of \( X^+ \) then \( A \subseteq X^- \) and the best alternative in \( A \) according to \( \succeq \) is the last element in \( A \) according to \( O \), which is \( c(A) \).

### 2.5 The money pump argument

The assumption that a choice function is rationalizable is sometimes defended on the ground that behavior that is inconsistent with rationality could produce choices that harm the individual.

Suppose that \( X \) consists of three alternatives, \( a, b, \) and \( c \), interpreted as objects, and that an individual’s choice function assigns \( a \) to \( \{ a, b \} \), \( b \) to \( \{ b, c \} \), and \( c \) to \( \{ a, c \} \). An implication of this choice function is that for any object \( x \), if the individual holds \( x \) then there is an object \( y \) such that the individual is willing to exchange \( x \) for \( y \); given that he prefers \( y \) to \( x \), he is willing to pay some (possibly small) amount of money to make the exchange. Assume that for each such exchange, this amount of money is at least $1. In this case, a manipulator could first give \( a \) to the individual, then offer to replace \( a \) with \( c \) in return for $1, then
offer to replace $c$ with $b$ in return for another $1$, and then offer to replace $b$ with $a$ for yet another $1$. After these three exchanges, the individual holds $a$, as he did initially, and is $3$ poorer. The manipulator can repeat the exercise, taking as much money from the individual as he likes. Such a mechanism is known as a money pump.

In fact, for any choice function $c$ that does not satisfy condition $\alpha$, such manipulation is possible. Assume that there are sets $A$ and $B$ with $B \subset A \subseteq X$ and $c(A) \in B$ and $c(B) \neq c(A)$. The manipulation goes as follows.

Take $c(A)$. (i) Are you willing to replace $c(A)$ with any element in $B \setminus \{c(A)\}$ for some amount of money? The individual can now choose from the set $B$ and will agree and choose $c(B)$. (ii) Are you willing to replace $c(B)$ with an alternative in $A \setminus \{c(B)\}$ for some amount of money? The individual can now choose from the entire set $A$ and will agree and choose $c(A)$. The manipulator can repeat the two steps as many times as he wishes.

The effectiveness of the manipulation depends on the inability of the manipulated individual to notice the exploitation. We leave it to you to judge whether the argument is a persuasive justification of the assumption that choice is rationalizable.

### 2.6 Evidence of choices inconsistent with rationality

Ample research demonstrates that human behavior is sometimes not rational in the sense we have defined. From the multitude of examples, we select three experiments that demonstrate this point; for each example, we identify features of behavior that are inconsistent with the assumption of rational behavior. The first experiment involves a situation in which some subjects’ choices conflict with property $\alpha$. The second and third experiments challenge the assumption that an individual chooses an alternative from a set, independently of the way the set is described. The experiments were first conducted many years ago (see the Notes at the end of the chapter). Here we report results of online experiments (using the website [http://gametheory.tau.ac.il](http://gametheory.tau.ac.il)) in which the subjects were a large number of students around the world with similar academic backgrounds to those of the potential readers of this book.

#### 2.6.1 Attention effect

Which of the following cameras do you choose?

- **Camera A**  Average rating 9.1, 6 megapixels
- **Camera B**  Average rating 8.3, 9 megapixels
Now make another choice.

<table>
<thead>
<tr>
<th>Camera A</th>
<th>Average rating 9.1, 6 megapixels</th>
</tr>
</thead>
<tbody>
<tr>
<td>Camera B</td>
<td>Average rating 8.3, 9 megapixels</td>
</tr>
<tr>
<td>Camera C</td>
<td>Average rating 8.1, 7 megapixels</td>
</tr>
</tbody>
</table>

Each question was answered by about 1,300 participants on the website http://gametheory.tau.ac.il. The results are given in the following tables.

<table>
<thead>
<tr>
<th>Choice between A and B</th>
<th>Choice between A, B, and C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Camera A</td>
<td>Camera A 30%</td>
</tr>
<tr>
<td>Camera B</td>
<td>Camera B 68%</td>
</tr>
<tr>
<td>Camera C</td>
<td>Camera C 2%</td>
</tr>
</tbody>
</table>

Thus the appearance of \( C \) does not lead people to choose \( C \), but rather causes a significant fraction of participants to choose \( B \), which dominates \( C \), even though in a choice between \( A \) and \( B \) they choose \( A \). One explanation of this result is that the availability of \( C \) directs the participants’ focus to \( B \), the alternative that dominates it. An alternative explanation is that the dominance of \( B \) over \( C \) provides a reason to choose \( B \), a reason that does not apply to \( A \).

### 2.6.2 Framing effects

Sometimes individuals’ choices depend on the way in which the alternatives are described.

You have to spin either roulette \( A \) or roulette \( B \). The outcomes of spinning each roulette are given in the following table.

<table>
<thead>
<tr>
<th></th>
<th>White</th>
<th>Red</th>
<th>Green</th>
<th>Yellow</th>
</tr>
</thead>
<tbody>
<tr>
<td>roulette A</td>
<td>90%</td>
<td>6%</td>
<td>1%</td>
<td>3%</td>
</tr>
<tr>
<td></td>
<td>$0</td>
<td>$45</td>
<td>$30</td>
<td>$-15</td>
</tr>
<tr>
<td>roulette B</td>
<td>90%</td>
<td>7%</td>
<td>1%</td>
<td>2%</td>
</tr>
<tr>
<td></td>
<td>$0</td>
<td>$45</td>
<td>$-10</td>
<td>$-15</td>
</tr>
</tbody>
</table>

Which roulette do you choose?

Subjects’ choices in this experiment are generally split more or less equally between the two roulettes. About 51% of around 4,000 participants at the website http://gametheory.tau.ac.il have chosen \( A \).
A common explanation for the choice of $A$ is that the problem is complicated and participants simplify it by “canceling” similar parameters. The outcomes of White in the two roulettes are identical and the outcomes of Red and Yellow are very similar; ignoring these colors leaves Green, which yields a much better outcome for roulette $A$.

Here is another choice problem.

You have to spin either roulette $C$ or roulette $D$. The outcomes of spinning each roulette are given in the following table.

<table>
<thead>
<tr>
<th></th>
<th>White</th>
<th>Red</th>
<th>Black</th>
<th>Green</th>
<th>Yellow</th>
</tr>
</thead>
<tbody>
<tr>
<td>roulette $C$</td>
<td>90%</td>
<td>6%</td>
<td>1%</td>
<td>1%</td>
<td>2%</td>
</tr>
<tr>
<td></td>
<td>$0</td>
<td>$45</td>
<td>$30</td>
<td>$15</td>
<td>$15</td>
</tr>
<tr>
<td>roulette $D$</td>
<td>90%</td>
<td>6%</td>
<td>1%</td>
<td>1%</td>
<td>2%</td>
</tr>
<tr>
<td></td>
<td>$0</td>
<td>$45</td>
<td>$45</td>
<td>$10</td>
<td>$15</td>
</tr>
</tbody>
</table>

Which roulette do you choose?

It is clear that $D$ dominates $C$, and indeed almost all participants (93%) at http://gametheory.tau.ac.il have chosen $D$.

Now notice that $A$ and $C$ differ only in their presentation (the color Yellow in $A$ is split in $C$ into two contingencies). The same is true of $B$ and $D$ (the color Red in $B$ is split in $D$ into two contingencies). The different presentations seem to cause at least half of the participants to apply different choice procedures: they reduce the complicated problem to a simpler one in the choice between $A$ and $B$ and apply a domination criterion in the choice between $C$ and $D$.

### 2.6.3 Mental accounting

Imagine that you have bought a ticket for a show for $40. When you reach the theatre you discover that you have lost the ticket. You can buy another ticket at the same price. Will you do so?

Now think about another situation.

Imagine that you intend to go to a show. When you take your wallet out of your pocket to pay for the $40 ticket, you discover that you have lost $40, but you still have enough cash to buy a ticket. Will you do so?
In both of these situations, you face a choice between
1. having $80 less than you did before departing home and seeing the perfor-
mance
2. having $40 less than you did before departing home and not seeing the per-
formance.

Although in both situations you face these same options, more people choose
to buy the ticket in the second situation than in the first situation. About 65%
of the 1,200 participants at http://gametheory.tau.ac.il have stated that
they would buy a new ticket in the first situation, in which they discover they
have lost a ticket they purchased previously. Among a similar number of differ-
ent participants, 79% have stated they would buy a ticket after discovering that
they had lost $40. The reason for the difference seems to be that in the first case
people follow a mental accounting process that counts the price of a ticket as
$80, and they regard that price as too high. In the second case, some people ap-
ppear to think about the loss of the $40 as unrelated to the issue of ticket purchase
and count the price of a ticket as only $40.

Problems

1. *Five choice procedures*. Determine whether each of the following five choice
functions over a set $X$ is rationalizable. If the answer is positive, find a pref-
erence relation that rationalizes the choice function. Otherwise, prove that
the choice function is not rationalizable.

   a. The set $X$ consists of candidates for a job. An individual has a complete
      ranking of the candidates. When he has to choose from a set $A$, he first or-
ders the candidates in $A$ alphabetically, and then examines the list from
      the beginning. He goes down the list as long as the new candidate is bet-
ter than the previous one. If the $n$th candidate is the first who is better
      than the $(n+1)$th candidate, he stops and chooses the $n$th candidate. If
      in his journey he never gets to a candidate who is inferior to the previous
      one, he chooses the last candidate.

   b. The set $X$ consists of $n$ basketball teams, indexed 1 to $n$. The teams par-
ticipate in a round robin tournament. That is, every team plays against
every other team. An individual knows, for every pair of teams, which one
wins. When he chooses a team from a set $A$, he chooses the one with the
largest number of wins among the games between teams in $A$. If more
than one team has the largest number of wins, he chooses the team with
the lowest index among the tied teams.
c. The set $X$ consists of pictures. An individual has in mind $L$ binary criteria, each of which takes the value 0 (the criterion is not met) or 1 (the criterion is met). Examples of such criteria are whether the painting is modern, whether the painter is famous, and whether the price is above $1,000. The criteria are ordered: criterion$_1$, criterion$_2$, . . . , criterion$_L$. When the individual chooses a picture from a subset of $X$, he rejects those that do not satisfy the first criterion. Then, from those that satisfy the first criterion, he rejects those that do not satisfy the second criterion. And so on, until only one picture remains. Assume that any two alternatives have a criterion by which they differ, so that the procedure always yields a unique choice.

d. An individual has in mind two numerical functions, $u$ and $v$, on the set $X$. For any set $A \subseteq X$, he first looks for the $u$-maximal alternative in $A$. If its $u$ value is at least 10, he selects it. If not, he selects the $v$-maximal alternative in $A$.

e. An individual has in mind a preference relation on the set $X$. Each alternative is either red or blue. Given a set $A \subseteq X$, he chooses the best alternative among those with the color that is more common in $A$. In the case of a tie, he chooses among the red alternatives.

2. Property of a choice function satisfying property $\alpha$. An individual has a choice function that satisfies property $\alpha$. Consider two sets, $A$ and $B$, such that $c(A) \in B$ and $c(B) \in A$. Prove that $c(A) = c(B)$.

3. Alternative proof of Proposition 2.3. Prove Proposition 2.3 by showing that any satisficing choice function satisfies property $\alpha$.

4. Variant of satisficing. An individual follows a procedure that differs from the satisficing procedure only in that if he does not find any satisfactory alternative then he goes back and examines all the alternatives and chooses the one for which $v(x)$ is highest. Show that the individual’s choice function satisfies property $\alpha$ and construct a preference relation that rationalizes it.

5. Path independence. Consider the following property of a choice function, called path independence:

$$c(A \cup B) = c(\{c(A), c(B)\}) \text{ whenever } A \cap B = \emptyset.$$  

That is, if the individual splits a choice set into two disjoint subsets, makes a choice from each subset, and then chooses between those two alternatives, he chooses the same alternative as he does when he chooses from the entire set.
a. Let \( c \) be a choice function that assigns to each set the best alternative according to some preference relation. Show that \( c \) is path independent.

b. Show that any choice function that is path independent is rationalizable (by showing it satisfies property \( a \)).

6. Caring up to a limit. An individual has in mind two numerical functions \( u \) and \( v \) defined on the set \( X \). Given a choice problem \( A \), he first looks for the \( u \)-maximal element \( x \) in \( A \). If \( v(x) \geq v^* \), he chooses \( x \). Otherwise, he chooses the \( v \)-maximal element in \( A \). (Notice that this choice function differs from the one in Problem 1d.)

a. Interpret the choice function in the case that \( u \) is a measure of the well-being of a friend and \( v \) is a measure of the wellbeing of the individual.

b. Show that for some \( X, u, \) and \( v \) the procedure is not rationalizable.

7. Extension of Proposition 2.2. Let \( X \) be an infinite set and \( c \) a choice function on \( X \). Show, using the following two steps, that if \( c \) satisfies property \( a \) then it can be rationalized.

a. Define a binary relation \( \succeq \) by \( x \succeq y \) if \( c(\{x, y\}) = x \). Show that this relation is a preference relation.

b. Show that for every choice problem \( A \), \( c(A) \succeq a \) for every \( a \in A \).

8. Money pump. Can a trader who thinks that \( 2 + 3 = 6 \) survive in our cruel world?

Notes

Property \( a \) was formulated by Chernoff (1954, Postulate 4, 429). The notion of satisficing is due to Simon (1956). The idea of a money pump appears to be due to Davidson et al. (1955, 145–146). Example 2.3 is taken from Luce and Raiffa (1957, 288). The experiment in Section 2.6.1 is based on the idea in Huber et al. (1982). The experiment in Section 2.6.2 was suggested by Tversky and Kahneman (1986, S263–S265). Section 2.6.3 is taken from Kahneman and Tversky (1984, 347–348). The exposition of the chapter draws on Rubinstein (2006a, Lecture 3).