Appendix 1: Reproduction Conditions

In volume 2 of *Capital,* Marx presents various schemes aimed at identifying the conditions of reproduction of a capitalist economy. Among other things, he provides numerical examples referring to a two-sector model, which, in the case of "simple reproduction", can be typified in the following way (Sweezy 1942, 162):

$$C_1 + V_1 + S_1 = V_1 + S_1 + V_2 + S_2$$

$$C_2 + V_2 + S_2 = C_1 + C_2$$

where C_i, V_i, S_i are the quantities of labour embodied in constant capital, variable capital and the surplus value of sector *i*.

To simplify, assume that each sector produces a single commodity, a consumer good (sector 1) and a capital good (sector 2). The above equations represent two conditions of equality between demand and supply. The left-hand sides are the values of supply, the right-hand sides are the values of demand. There are no net investments, and the workers' incomes, V_1 and V_2, as well as the capitalists' incomes, S_1 and S_2, are entirely spent in buying the consumer good. C_1 and C_2 are the parts of revenues spent by capitalists to replace the advances of capital.

Reproduction of the system implies:

$$C_1 = V_2 + S_2$$

meaning that the revenue spent by the first sector capitalists to buy the capital good must be equal to the income spent by the second sector workers and capitalists to buy the consumer good.

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Reproduction conditions pertain to the *physical* consistency of the production structure: the two sectors must produce the quantities of commodities required to replicate production itself. Marx seems to think that, since equality of the demand and supply of a commodity can be expressed in physical terms, it does not matter whether their magnitudes are defined in labour values or in monetary prices: "the fact that prices diverge from values cannot, however, exert any influence on the movement of the social capital. On the whole, there is the same exchange of the same quantities of products" (Marx 1997, 392).

Yet demand and supply are decided in a market system where production decisions are motivated by the profit goal, and not by the aim of satisfying social needs. What is to be determined, therefore, is the set of *exchange values* that grant reproduction. In other words, reproduction conditions are about the prices that ensure the perpetuation of the technical and social structure of production. This refers to not only the reproduction of commodities, but also "the reproduction (i.e. maintenance) of the capitalist class and the working class, and thus the reproduction of the capitalist character of the entire process of production" (Marx 1997, 391). Not all values are appropriate, and certainly not labour values, as I will show in a moment.

A competitive *reproduction equilibrium* is the state of a capitalist economy in which markets clear and profit rates are uniform. It is achieved through a process in which: a) market prices and profits change in response to excess demands, b) supplies and demands of commodities change as consequences of consumption and investment decisions. The latter are driven by the capitalists' quest for high profits:

> The competition between capitalists–which is itself this movements towards equilibrium–consists here of their gradually withdrawing capital from spheres in which profit is for appreciable length of time below average, and gradually investing capital into spheres in which profit is above average (Marx 1998, 364).

It goes without saying that equilibrium can only occur by chance. Yet it represents the state of an economy toward which market prices and the actual profit rates should tend to gravitate:

> The different spheres of production […] constantly tend to an equilibrium: for, on the one hand, while each producer of a commodity is bound to produce a use value, to satisfy a particular social want, and while the extent of these wants differs quantitatively, still there exists an inner

> relation which settles their proportion into a regular system […]; and, on the other hand, the law of value of commodities ultimately determines how much of its disposable working time society can expend on each particular class of commodities (1996, 361).

The profit rate is brought about by market competition, but not determined by market forces:

> The average rate of profit sets in when there is an equilibrium of forces among the competing capitalists. Competition may establish this equilibrium but not the rate of profit which makes its appearance with this equilibrium (852).

The uniform profit rate is determined by production conditions, and works as an incentive to replace the capital advances that warrant reproduction of the industrial system. It is a measure of investment returns that induces capitalists to plan the required proportions of activity levels.

In reproduction equilibrium, commodities exchange at production prices. So, let me reshape the above equations in the following way (Screpanti 1993, 9):

$$p_1q_1 = wl_1q_1 + wl_2q_2 + p_2(a_{21}q_1 + a_{22}q_2)r \quad \text{(A1)}$$

$$p_2q_2 = p_2(a_{21}q_1 + a_{22}q_2) \quad \text{(A2)}$$

In this model, q_1 and q_2 are the quantities produced of the consumer and the capital goods, p_1 and p_2 are their *monetary* prices, w is the nominal wage, a_{21} and a_{22} are the technical coefficients in sectors 1 and 2, l_1 and l_2 are the labour coefficients, and r is the rate of profit. All symbols represent scalars. The left-hand sides of the two equations are the values of supplies, the right hand sides the values of demands.

It must be

$$(p_1 - wl_1 - p_2a_{21}r)q_1 = p_2(1 - a_{22})q_2 \quad \text{(A3)}$$

This means that the value of the consumer good not consumed by the workers and the capitalists in the consumer good sector has to be equal to the value of the capital good not consumed by the capitalists in the capital good sector.

Equations (A1) and (A2) can be rewritten

$$(p_1 - wl_1 - p_2a_{21}r)q_1 = (wl_2 + p_2a_{22}r)q_2$$

$$p_2(1 - a_{22})q_2 = p_2a_{21}q_1$$

which, substituting from (A3), become

$$p_2(1 - a_{22})q_2 = (wl_2 + p_2a_{22}r)q_2 \qquad \text{(A1')}$$

$$(p_1 - wl_1 - p_2a_{21}r)q_1 = p_2a_{21}q_1 \qquad \text{(A2')}$$

The two equations entail

$$1 + r = \frac{p_2 - wl_2}{p_2a_{22}} = \frac{p_1 - wl_1}{p_2a_{21}}$$

These are the *conditions of reproduction*. Given the wage, they determine the uniform profit rate and the production prices that ensure market clearing.

Notice that they could be obtained more directly from equation (3) of chapter 4, which, under the assumptions of this appendix, can be restyled as

$$p_1 = wl_1 + p_2a_{21} + p_2a_{21}r \equiv V_{p1} + C_{p1} + S_{p1}$$

$$p_2 = wl_2 + p_2a_{22} + p_2a_{22}r \equiv V_{p2} + C_{p2} + S_{p2}$$

where V_{pi}, C_{pi} and S_{pi} are the monetary expressions of variable capital, constant capital and surplus value in sector *i*.

Labour values are defined as

$$v_1 = w_vl_1 + v_2a_{21} + (1 - w_v)l_1 \equiv V_1 + C_1 + S_1$$

$$v_2 = w_vl_2 + v_2a_{22} + (1 - w_v)l_2 \equiv V_2 + C_2 + S_2$$

It is evident that, except in the case of a uniform organic composition of capital, conditions $V_{pi}=V_i$, $C_{pi}=C_i$, and $S_{pi}=S_i$ only occur when $r=0$, $p_1=v_1$, $p_2=v_2$ and $w=w_v$. When $r>0$, conditions $p_1/p_2 \neq v_1/v_2$ and $w \neq w_v$ hold generically, and therefore $V_{pi} \neq V_i$, $C_{pi} \neq C_i$, and $S_{pi} \neq S_i$. The law of value conservation, which is not valid in the overall aggregate, a fortiori does not apply in the sectorial aggregates. In other words, reproduction conditions in a capitalist economy cannot be determined in labour values.